

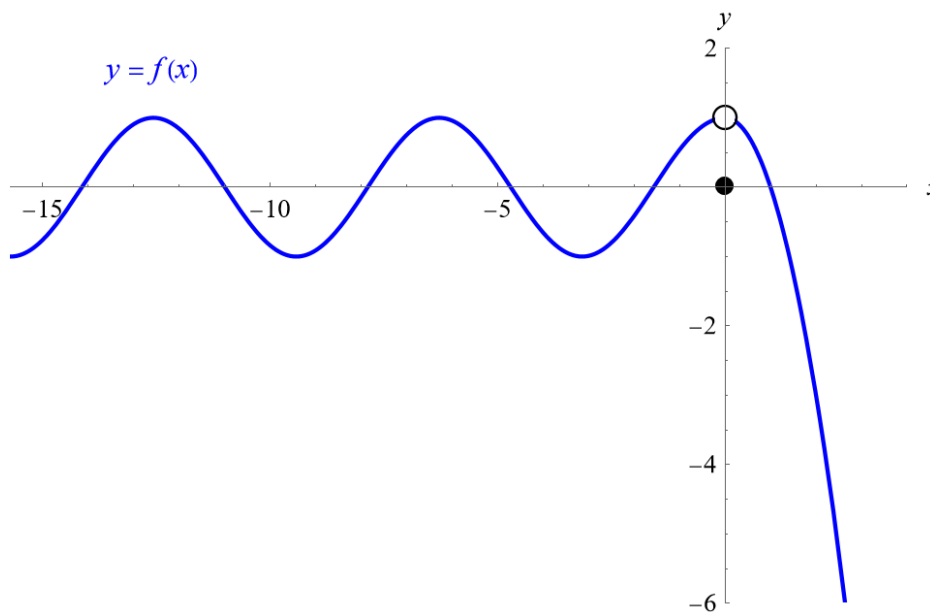
Exercise 21

Explain why the function is discontinuous at the given number a . Sketch the graph of the function.

$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases} \quad a = 0$$

Solution

A graph of the function versus x is shown below.



The function is discontinuous at $x = 0$ because although the left-hand and right-hand limits are both equal to 1 there, they are not equal to the value of the function there, which is 0.

$$\lim_{x \rightarrow 0} f(x) = \cos 0 = 1 - 0^2 = 1 \neq f(0) = 0$$